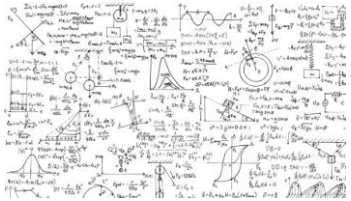
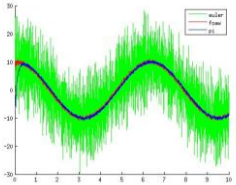

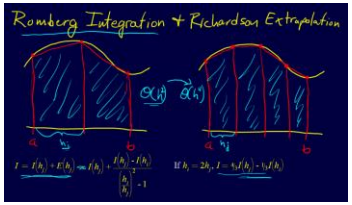

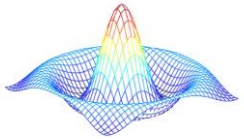

MINE 2002 Engineering Mathematics

2018 – 2019 Spring

Week 5 (Roots of Equations)

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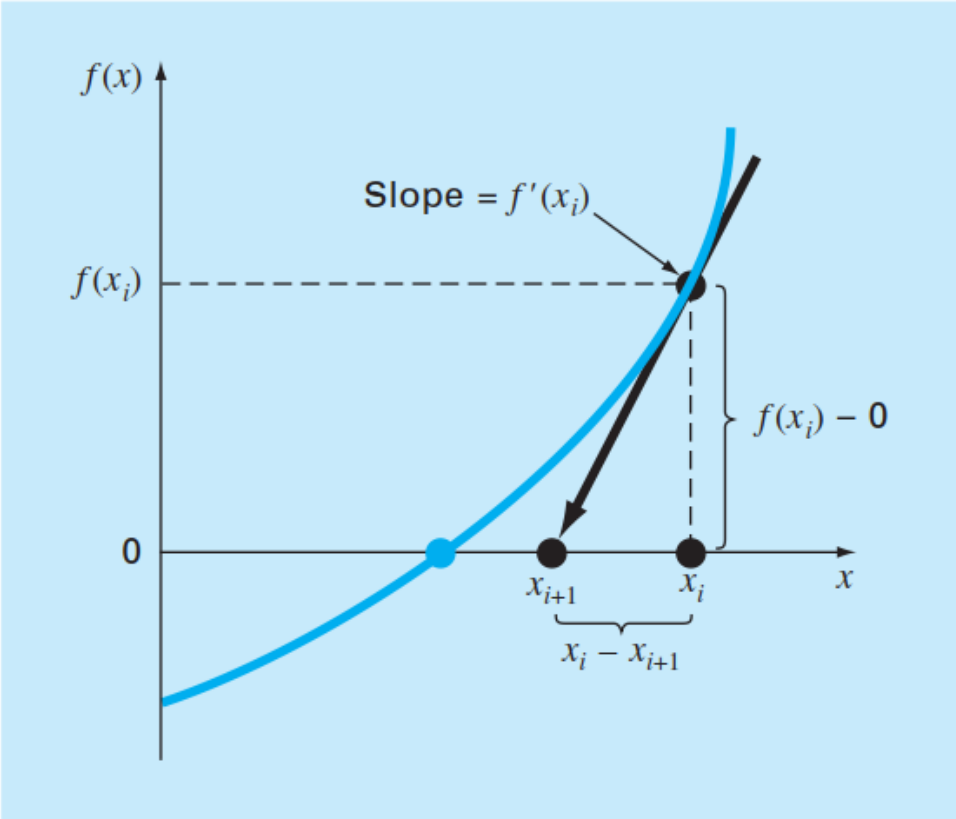
Course Outline

Week	Date	Course Content
1		Warming up, general introduction
2		Approximation, round-off errors, truncation errors
3		Roots of equations: Closed-form methods
4		Roots of equations: Fixed-point iteration
5		Roots of equations: The Newton-Raphson method
6		Roots of equations: The secant method
7		Roots of polynomials and general review
8		Midterm exam week
9		Linear algebraic equations: Gauss elimination
10		Linear algebraic equations: Gaus-Seidel method
11		Curve fitting: Least-squares regression
12		Interpolation
13		Numerical integration
14		Numerical differentation and general review
15		Final exam week

Roots of Equations: Newton-Raphson Method

Perhaps the most widely used of all root-locating formulas is the Newton-Raphson equation. If the initial guess at the root is xi, a tangent can be extended from the point [xi, f(xi)]. The point where this tangent crosses the x axis usually represents an improved estimate of the root.



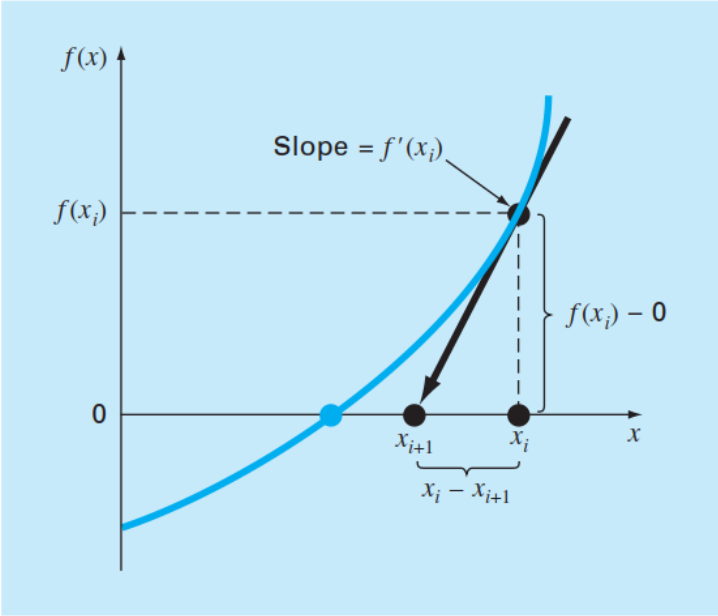
Roots of Equations: Newton-Raphson Method

The Newton-Raphson method can be derived on the basis of this geometrical interpretation. The first derivative at x is equivalent to the slope:

$$f'(x_i) = \frac{f(x_i) - 0}{x_i - x_{i+1}}$$

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

which is called the Newton-Raphson formula



Roots of Equations: Newton-Raphson Method

Numerical Example:

Use the Newton-Raphson method to estimate the root of

$$f(x) = e^{-x} - 1$$

employing an initial guess of x0 = 0.

Roots of Equations: Newton-Raphson Method

Numerical Example:

$$f'(x) = -e^{-x} - 1$$

$$x_{i+1} = x_i - \frac{e^{-x_i} - x_i}{-e^{-x_i} - 1}$$

i	x_i	ε_t (%)
0	0	100
1	0.500000000	11.8
2	0.566311003	0.147
3	0.567143165	0.0000220
4	0.567143290	$< 10^{-8}$

Roots of Equations: Newton-Raphson Method

Numerical Example:

Find the root of the below given function starting from x = 0.5.

$$f(x) = e^x - 5\sin(\frac{x\pi}{2})$$

Roots of Equations: Newton-Raphson Method

Numerical Example:

Find the root of the below given function starting from x = 0.5.

$$f(x) = e^x - 5\sin(\frac{x\pi}{2})$$

x_o	$f(x_o)$	$f'(x_o)$	$x=x-f(x)/f'(x)$
0.5000	-1.8879	-3.9054	0.01657989
0.0166	0.8865	-6.8378	0.14622213
0.1462	0.0186	-6.4932	0.14909410
0.1491	0.0000	-6.4817	0.14909664

Roots of Equations: Newton-Raphson Method

Numerical Example:

Find the root of the below given function starting from x = 1.5. Let the εa < 0.5 %.

$$f(x) = x^6 - x - 1$$

Roots of Equations: Newton-Raphson Method

Numerical Example:

Find the root of the below given function starting from x = 1.5. Let the εa < 0.5 %.

$$f(x) = x^6 - x - 1$$

n	x_n	$f(x_n)$
0	1.5	8.890625
1	1.30049	2.54
2	1.18148	5.38E-1
3	1.13945	4.92E-2
4	1.13477	5.50E-4
5	1.13472	7.11E-8
6	1.13472	1.55E-15

Roots of Equations: Newton-Raphson Method

In-Class-Application:

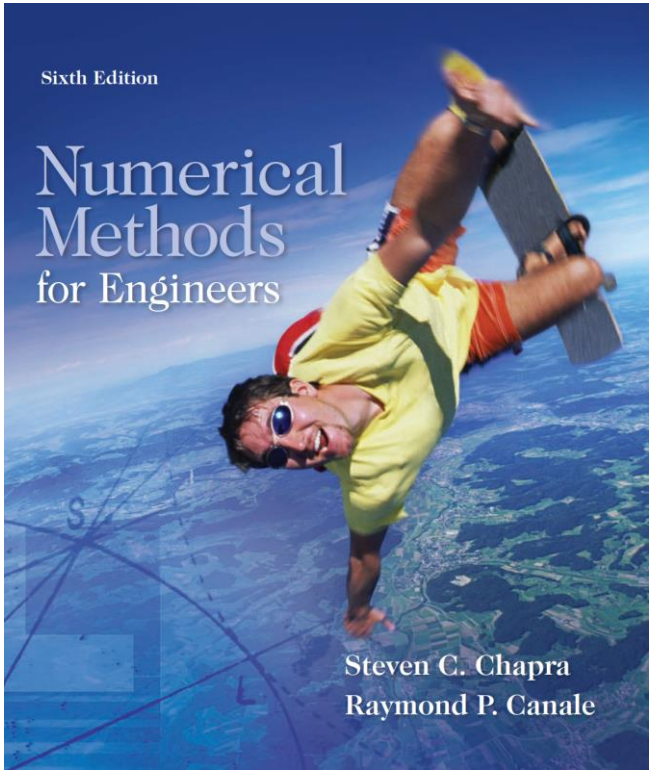
Find the root of the below given functios. Let the εa < 0.5 %.

$$f(x) = e^x - 3x \qquad x_0 = 0.5$$

$$f(x) = x^3 + 2x^2 - 3x - 1 \qquad x_0 = 1$$

Essential Vocabulary:

Course reference



End of the lecture...

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